Coloring-based Resource Allocations in Ad-hoc Wireless Networks

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***Abstract*— It is well known that CSMA/CA protocols exhibit very poor performance in case of multi-hop transmissions, because of inter-link interference due to imperfect carrier sensing. We propose to control such an interference by pre-allocating temporal slots in which different sets of network nodes are allowed to contend for the channel access. The approach is based on distributed coloring algorithms with limited signaling overhead that can be customized as a function of the network topology and traffic load.**

I. Introduction

Ad-hoc networks are suitable for a large number of applications, spanning from low-range sensor networks targeted to distributed monitoring, to high-range mesh networks targeted to build infrastructure-less transport networks. Regardless of the specific physical layer technology (such as IEEE 802.15.4 PHY or 802.11a/b/g/n PHY, defining available bandwidth, transmission power, modulation/coding schemes, and so on), most ad-hoc networks rely on contention-based medium access protocols, since the use of carrier sense and random backoff mechanisms is a simple and well-established solution for distributedly managing multiple-access over a shared channel bandwidth. However, it is well known that CSMA/CA protocols exhibit very poor performance in case of multi-hop transmissions. Theoretical bounds on the attainable limits of throughput in presence of imperfect carrier sensing have been studied. In the seminal paper [1] bounds were determined for a network with arbitrarily or randomly deployed nodes under the assumption that an ideal scheduling scheme for arbitrating node transmissions can be implemented. More recently, in [2] some analysis extensions have been considered, for quantifying the impact of mobility and node cooperation on such bounds. The hidden terminal problem of CSMA/CA protocol is addressed in many papers as [3]. Moreover, in [4] different time-division scheduling for ad-hoc networks, with an analysis of the TDMA policy is presented.

This paper deals with the distributed resource allocation problems for multi-hop wireless networks. The basic idea is combining the TDMA approach (for grouping the contending nodes in non-interfering sets) with the CSMA/CA approach (for managing the final access to the shared channel). Similar solutions have been considered in recent standard extensions (such as 802.11s) and literature, for optimizing both the network capacity and the energy consumption [5] in Zigbee networks, or coping with bidirectional traffic flows over chain topologies exploiting network coding [6]. The proposed solutions consist in scheduling potentially interfering transmissions in different time slots, while allowing in-range nodes to transmit in the same time slot but subject to a CSMA/CA mechanism that avoids collisions.

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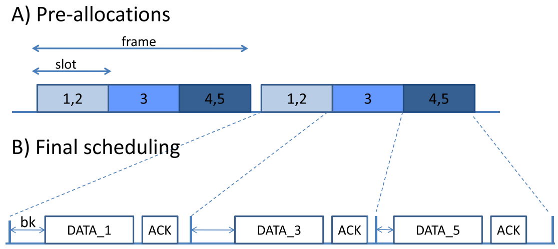

Fig. 1. An example of medium access in a network with 5 nodes and a frame composed of 3 slots.

In this paper, we focus on the problem of determining the best number of slots in a frame and the best assignment of slots to different links. The problem is formulated in terms of a map coloring problem, which has a vast and well established literature [7], [8]. Therefore, we simply adapt some existing coloring algorithms to ad-hoc networks, by trying to identify the most effective trade-offs between complexity, signaling overheads and performance gain.

II. Problem Formulation

We consider a single channel radio network made of set V of nodes distributed uniformly over a given area. Each node $i \in V$ can communicate only with a subset of adjacent nodes V_i. We say that i is (radio) *visible* only to the nodes in V_i. Differently, i is *hidden* to the remaining nodes in $V \setminus V_i$. We assume that radio visibility is symmetric and that the communication between pair (i, j) of adjacent nodes presents a maximum transmission rate r_{ij}, function of the distance between nodes i and j and of the possible presence of obstacles.

The channel time is divided into elementary allocation units called slots. Each slot is able to accommodate a random backoff delay and the transmission time of the maximum allowed packet size at the minimum rate (followed by an explicit acknowledgment). Only the subset of nodes to which a generic channel slot is pre-allocated are enabled to perform the CSMA/CA function for transmitting on that slot. The slot allocations are maintained on a per-frame basis: being x the total number of allocation slots, a sequence of x consecutive slots is a channel frame in which, slot by slot, the same sorted list of nodes are enabled to transmit. Figure 1 shows an example of medium access in a network with 5 nodes, in which a channel frame of 3 slots is considered. In the first slot, where only station 1 and 2 can access the medium, station 1 wins the contention (i.e. extracts the lowest backoff delay). The second slot is used by station 3 only, while the third slot is reserved to the contention between stations 4 and 5. The reason for pre-allocating channel slots to a sub-set of stations (thus grouping in independent sets the stations allowed to transmit simultaneously) is the mitigation of the hidden node problem. For example, if stations 1 and 3 are hidden to each other and wish to transmit to station 2 (which is able to hear both the stations), the previous

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allocation avoids any collision possibility. We formally define the problem of slot allocations in what follows.

Network Structure and Traffic Model

We represent the network structure through an edge labeled graph $G = (V, E)$. Specifically, the nodeset V includes all the nodes i of the network and the edgeset E includes all the pairs of adjacent nodes (i, j) that are in radio visibility to each other. Each edge $(i, j) \in E$ is labeled with its maximum transmission rate r_{ij}.

Since the system time is slotted, we also model the traffic source at each node in terms of per-slot packet probability. Specifically, we assume that each node i has a fixed probability λ_i to generate a packet in each slot. In order to avoid interactions with the routing protocol, we consider only one-hop packet deliveries. Packets are destined to a randomly selected node among the neighbor ones. For isolated nodes, i.e. nodes without neighbors, the traffic is assumed to be broadcast. In addition, we assume that the packet size is of a fixed value D for all the nodes, whose transmission time is always compatible with the slot size.

Resource Allocation

We assume that the reader is familiar with CSMA/CA protocols, which regulate the final channel access within an allocated channel slot. Although most CSMA/CA protocols use a slotted backoff scale for efficiency reasons and for implementation limits (since the carrier sense cannot be instantaneous), we assume that backoff values are uniformly extracted in a continuous range $[0, b]$, thus implying that collisions cannot be originated by the extraction of two identical backoff values.

In order to implement a slot allocation mechanism, two basic functionalities have to be provided in the network: i) a mechanism for inferring the network topology; ii) a mechanism for keeping a common time reference among the nodes. For both the aspects, we consider that an independent signaling channel is available (managed by a random access scheme) and nodes in radio visibility can exchange control information (e.g. the list of neighbor nodes). We also assume that nodes do not have data storage constraints, while processing capabilities may depend on the specific network scenario.

In the above context, the two main problems of our interest are the following ones.

Problem 1: Determine a distributed protocol that sets the number x of slots in a frame and the slots allocations, in order to maximize the per-node throughput in saturation conditions, i.e. in presence of greedy sources whose packet generation rate is $\lambda_i = 1$.

Problem 2: Determine a distributed protocol that allows the allocations of slots of a frame in order to minimize the average delivery delay for generic source rates λ_i.

III. Solution Approach

Here, we discuss the possibility of reducing Problems 1 and 2 to a set of Minimum Graph Coloring (MGC) problems.

Graph coloring

Let $G(V, E)$ the network graph including the set V of nodes distributed over a given area and the set E of edges connecting radio visible nodes. Each node $i \in V$ is labeled with the number a_i of slots to be allocated to it according to the traffic it must support. Generally speaking a single slot is allocated to each node. However, in case of heterogeneous packet generation rates λ_i (which may actually model nodes belonging to heterogeneous number of paths and aggregating traffic packets generated by multiple sources), some nodes may require more slots to drain their traffic.

We define as *incompatibility graph of type I* the node labeled graph $H_E(V, F_E)$ whose edges in F_E join the pair of nodes $(j, k) \in V \times V$ whose frames may collide if transmitted simultaneously. By definition $H_E = G^2$, that is,

$$F_E = \{(j, k) : \exists i \in V \text{ s.t. } (j, i), (i, k) \in E\}.$$

We can see Problems 1 and 2 as a MGC problem that determines the minimum cardinality of a coloring of the nodes of H_E such that each node is colored with as many different colors as its label. Then, each color corresponds to a specific slot allocated to the node on the frame.

Obviously, the network transport capacity is critically affected by the cardinality x of a coloring of H_E, since each node i receives a_i transmission chance only every x slots. For example, assuming a uniform transmission rate r among all the edges, the node transmission rate is upper bounded by $a_i \cdot r/x$.

In defining the incompatibility graph of type I, we have not considered the carrier sense functionality that intrinsically makes orthogonal (i.e. non-interfering) the transmissions between visible nodes. Edges connecting visible nodes in the incompatibility graph of type I may result redundant and some, if not all of them, may be removed, possibly drastically reducing the number of colors necessary for the graph. In this context, we define as *incompatibility graph of type II* the node labeled graph $H_\emptyset(V, F_\emptyset)$ whose edges in F join the pair of nodes $(j, k) \in V \times V$ that are of the reciprocally hidden and whose frames may collide if transmitted simultaneously. By definition $H_\emptyset = G^2 - G$, that is,

$$F_\emptyset = \{(j, k) : \exists i \in V \text{ s.t. } (j, i), (i, k) \in E \text{ but } (j, k) \notin E\}.$$

Removing edges from the incompatibility graph can make the per-node transmission rate S_i (also called node throughput) heterogeneous, even in the case of uniform rate r and a_i allocations. Indeed, nodes receiving slots not shared with visible nods receive a throughput bounded by $a_i \cdot r/x$, while nodes sharing the slot with neighbor nodes experience, in the worst case, a throughput reduction equal to the number of contending nodes.

The graph H_E and $H_\emptyset$ define the two extreme cases in which either all or none of the pair of reciprocally visible nodes are considered. Obviously, even intermediate situations may be defined. Let 2^E be the power set of the edgeset E.

For each $e \in 2^E$, we can consider the coloring problems of the incompatibility graphs $H_e = (V, F_\emptyset \cup e)$, and the per-node and aggregated throughput, S_i^e and $S_{tot}^e = \sum_{i \in V} S_i^e$. Then, the optimal coloring scheme is the coloring referring to the incompatibility graph H_e which maximizes the value of S_{tot}^e, for $e \in 2^E$.

Throughput assessment

Consider a H_e, for $e \in 2^E$, graph. For each node $i \in V$, let us define its *associated after coloring clique* as the maximal clique on the graph G that includes i and is formed by nodes of the same color of i. Let d_i^e be the size of such a clique and let $a_i = 1 \ \forall i$.

Then, if we assume a uniform rate r for all the links in

E, we can guarantee a per-node *collision free throughput*

$$\rho_i^e = \frac{r}{x^e d_i^e} \tag{1}$$

where, x^e is the number of colors used in H_e. The rationale behind (1) is the following. For each node $i \in V$, we have to share the slot associated to its allocated color with $d_i^e - 1$ contending nodes. On average, node i will win the backoff contention only once every d_i^e frames. Collisions with adjacent nodes are avoided by means of the carrier sense functionalities, while collisions with other nodes using the same colors are avoided by the coloring algorithm (which re-assign the same slot only when nodes are distant more than two hops).

Given a graph H_e, the maximum number of needed colors is upper bounded by $\Delta^e + 1$, where Δ^e is the maximum node degree of the graph. In addition, coloring H_e with at maximum $e + 1$ colors can be easily attained with a distributed protocol, such as *Brooks-Vizing*, [11]. The following condition then holds:

$$\frac{r}{\Delta^E+1} \quad \geq \quad \max_{i\in V} \frac{r}{(\Delta^e+1)d_i^e} \quad \forall e \in 2^E \tag{2}$$

Since Δ^e is un upper bound on the number of needed colors, the previous condition implies that the lower bound of the collision-free throughput guaranteed to each node is higher for the incompatibility graph H_E.

Let us now consider the aggregated collision-free throughput ρ_{tot}^e. After coloring, the throughput sum perceived by all the nodes belonging to each clique is obviously r/x^e, thus resulting in a total throughput equal to:

$$\rho_{tot}^e = \frac{r}{x^e} c^e$$

where c^e is the total number of cliques resulting from the coloring of the incompatibility graph H_e. Obviously, when $H_e = H_E$ such a number corresponds to the number of nodes n (since 1-hop nodes are allocated on different channels). It follows that we can also express the average per-node throughput $E[\rho^e]$ as $\rho_{tot}^e/n = \frac{r}{x^e E[d^e]}$ (where $E[d^e] = n/c^e$ represents the average after coloring clique size).

For each $e \in 2^E$, we note that the collision free throughput is only a guaranteed lower bound on the actual throughput S_{tot}^e that we can obtain coloring the graph H_e. In fact, S_{tot}^e can be a higher throughput. Let x^e be the number of colors used for H_e. Consider a generic node i and let x_i the number of colors used for coloring its adjacent nodes on H_e. When $x_i < x^e + 1$, we may obtain a transmission rate for i greater than the one guaranteed by the collision free throughput if we allow i to transmit during the slots associated to its color and to colors different form the ones of its adjacent nodes on H_e. If i is the only node with such a privilege, we will obtain a throughput (with no collision) higher than the collision free throughput. Differently, if we concede the above transmission privilege to all the nodes with $x_i < x^e + 1$, we cannot guarantee collision free transmission any more. Nevertheless, we obtain an overall throughput higher than the collision free throughput, as each node during the slots associated to its own color is guaranteed that its transmission cannot occur in frame collisions.

Let us assume that we can tolerate some collisions or the overhead associated to the channel sensing. Then, the above argument suggests that the coloring emerging from the incompatibility graph of type I may be not optimal in term of throughput. As extreme example, consider the case in which the source activation rate is extremely low, close to zero. It is apparent that the slot partition of the channel is useless and not optimal.

Coloring Algorithms Coloring algorithms have been widely explored In literature. Some examples of popular solutions are the *Luby*'s algorithm [9], the *Johanson*'s algorithm [10], and their variants [11].

We consider an adaptation of the algorithm proposed in [11] and a simple extension of such a scheme. A preliminary exchange of control information is necessary for evaluating at each node i the global degree of the network Δ or the local number of neighbors δ_i. Let x_{max} the global or the local maximum number of available colors. According to the basic algorithm, each uncolored node has to perform the following steps:

1. *First coloring* Randomly pick a color from a list of available colors.
2. *Conflict Resolution* If none of your (1-hop or 2-hop) neighboring nodes has chosen the same color, keep it as definitive color, otherwise remove it form the list and try again the next step.
3. *List update* If the color list is empty, add new colors. The list is updated starting from $\min\{c{+}1, x_{max}\}$ color, where $c = \max\{\text{neighboring node colors}\}$.

We call this algorithm as *SC* algorithm, for recalling its characteristic of first *Selecting* a color and then *Comparing* the selected color with potential interferes.

In order to optimize the number of used colors, we also considered a simple modification of the previous scheme. Instead of randomly picking a color from the available ones, each node first updates the list of available colors (as in the third step of the previous scheme) and then selects the color with the lowest index. We call this scheme as *CFA* algorithm, since it is based on *Choosing* the *First Available* color.

IV. Numerical Results

In order to compare the effectiveness of the slot pre-allocations in improving the CSMA/CA performance in multi-hop networks, we run several simulations, including both the network coloring phase and the data transmission phase. Obviously, the throughput performance perceived in a given network topology are critically affected by the final map of colors and by the node source rates. For the same network topology, such a final map depends on the random color selections and/or on the node initialization choices. Therefore, each run performance can be different and has to be averaged. Note also that in our simulations, we do not consider dynamic node activations and de-activations, thus running the coloring phase only at the beginning of the simulation and maintaining the color map for the rest of the simulation time.

We considered random network topologies of 30 nodes deployed over an area of $10 \cdot 10m^2$, with a transmission range of $3m$. We observed that the CFA scheme requires on average 15 different colors when the incompatibility graph is $H_E = G^2$, while it uses 8 colors only for the graph $H_\emptyset = G^2 - G$. Conversely, the SC coloring scheme resulted in an average number of colors equal to 24 for the $G^2 - G$ case, and 17 for the $G^2 - G$ case. The higher number of adopted colors has two different effects: on one side it increases the frame length, thus resulting in a lower rate of node transmission

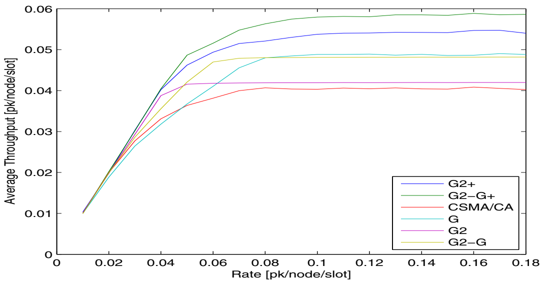

Fig. 2. Average throughput under the SC coloring scheme, for different incompatibility graphs and comparison with standard CSMA/CA.

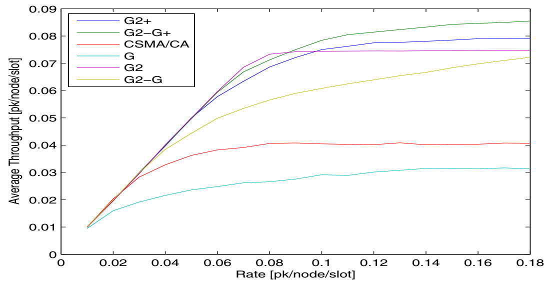

Fig. 3. Average throughput under the CFA coloring scheme, for different incompatibility graphs and comparison with standard CSMA/CA.

chances; on the other side it reduces the contention level between 1-hop nodes in case of $G^2 - G$.

After that each node has been colored, we simulated 5000 channel slots. At each slot, three different steps are considered: i) generation of traffic packets, ii) selection of transmitting nodes; iii) verification of transmission outcomes. At the first step, a new packet is generated in the transmission buffer of each node i with a fixed probability $\lambda_i = Rate$ $\forall i$. The destination node is uniformly extracted among the neighbors and no buffer size limit is considered. At the second step, the simulator processes all the nodes whose color corresponds to the current slot and extracts a backoff value for resolving potential contentions. All nodes winning the contention are labeled as transmitting. Finally, if the neighbors of the intended receiver are not transmitting, the transmission is considered successful and the packet is removed from the buffer. Otherwise, the packet remains in the buffer until a maximum number of retries (set to 3) has been reached.

Figures 2 and 3 compare the per-node average throughput measured in our simulations, under the SC and CFA scheme, for different incompatibility graphs (namely, G, G^2 and $G^2 - G$). In both the figures we also plot the CSMA/CA performance. The throughput has been averaged by considering ten different coloring runs of the coloring schemes, referring to the same network topology. From the figures we can draw some interesting observations. First, coloring G can be useless, because the carrier sense functionality is already able to avoid interference among adjacent nodes. For the CFA case, the performance obtained under the G coloring are even worse than the ones obtained with the CSMA/CA protocol, because the slot allocations may synchronize hidden nodes for lower packet generation rates. Second, coloring G^2 can be more efficient (CFA case) or less efficient (SC case) than coloring $G^2 - G$, according to the network topology and to the effectiveness of the coloring scheme in selecting a limited number of colors and/or leaving a limited number of bottlenecks. Third, when additional channel slots are allocated as described in section III-B (the G^2+ and G^2-G+ curves of the figures), the network throughput performance can be further improved.

Finally, to validate the throughput bounds discussed in section III-B, table I compares the saturation (collision-free) theoretical bounds with the best throughput values measured (on a given topology) under 10 different CFA coloring runs.

Topology	H_e	x_e	c^e	$\hat{\rho}^e$/r	$E[\rho^e]$/r
1	G^2	16	30	0.0624	0.0625
1	$G^2 - G$	5	12	0.0792	0.0800
2	G^2	13	30	0.0768	0.0769
2	$G^2 - G$	5	11	0.0730	0.0733
3	G^2	15	30	0.0666	0.0667
3	$G^2 - G$	5	13	0.0863	0.0867

TABLE I
MEASUREMENTS AND ESTIMATES OF THROUGHPUT.

V. CONCLUSIONS

Coordination among nodes in ad-hoc networks can significantly improves the transport capacity of the networks, in comparison with simple uncoordinated CSMA/CA protocols. A simple form of coordination can be provided by pre-allocating temporal intervals in which different sets of nodes are allowed to access the shared wireless medium. We have analyzed different solutions introducing such a pre-allocation on the basis of a neighbor discovery protocol and distributed coloring schemes requiring limited signaling overheads. We showed that the performance of these schemes can be critically affected by the considered incompatibility graph, trading off the contention-level experienced by 1-hop neighbors and the orthogonality guaranteed to hidden nodes.

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